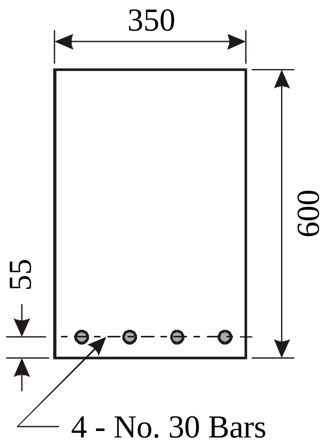
Question 1:



- Beam dimensions:

$h := 600 \cdot mm \quad b := 350 \cdot mm$

$cc := 55 \cdot mm$

$d := h - cc \qquad d = 545\,mm$

- Tension Steel:

$d_b := 30 \cdot mm \quad n_{bar} := 4$

$A_s := A_{s\_bar}(d_b) \cdot n_{bar} \qquad A_s = 2800\,mm^2$

- Material: $f'_c := 35 \cdot MPa \quad f_y := 400 \cdot MPa \quad \varepsilon_{cu} := 0.0035 \quad E_s := 200000 \cdot MPa$

Whitney stress block parameters: $\alpha_1 = 0.797 \quad \beta_1 = 0.882$

Concrete modulus of elasticity: $E_c := \left(3300 \cdot \sqrt{f'_c \cdot MPa} + 6900 \cdot MPa\right) \cdot \left(\dfrac{\gamma_c}{2300 \cdot \dfrac{kg}{m^3}}\right)^{1.5} \qquad E_c = 28165\,MPa$

Part a):

- Location of the neutral axis: Assume steel yields $f_s := f_y$

$\Sigma F_x = 0 \qquad C_c = T$

$C_c = (\alpha_1 \cdot f'_c) \cdot (a \cdot b) \qquad T := f_s \cdot A_s$

$(\alpha_1 \cdot f'_c) \cdot (a \cdot b) = f_s \cdot A_s \qquad a := f_s \cdot \dfrac{A_s}{(\alpha_1 \cdot f'_c \cdot b)} \qquad a = 114.6\,mm$

$c := \dfrac{a}{\beta_1} \qquad c = 129.9\,mm$

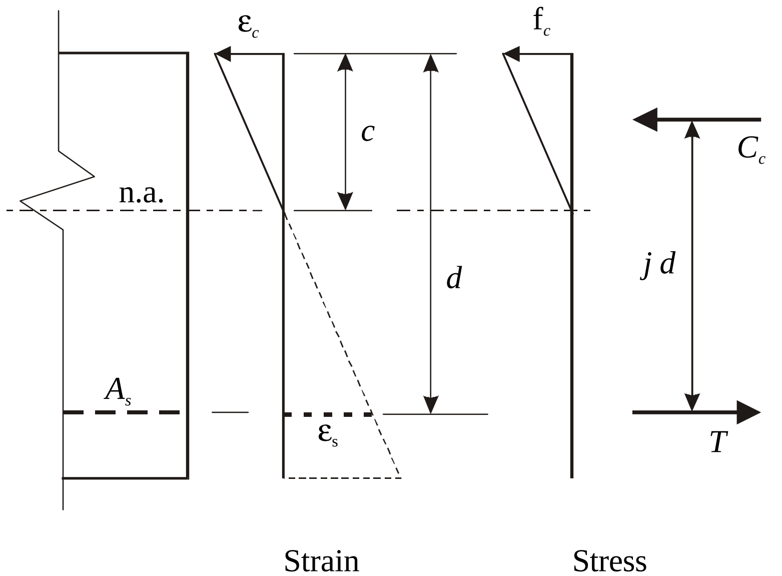
- Check for steel yielding

$\varepsilon_s := \varepsilon_{cu} \cdot \left(\dfrac{d - c}{c}\right) \qquad \varepsilon_s = 0.011 \quad > \varepsilon_y$ - OK

- Nominal Resisting Moment: $\Sigma M_{Cc} := 0$

$M_n := f_s \cdot A_s \cdot \left(d - \dfrac{a}{2}\right) \qquad M_n = 546.2\,kN \cdot m$

Part b): Linear stress-strain relationship in concrete



$\varepsilon_s := 0.00075$

$f_s := \varepsilon_s \cdot E_s$

$f_s = 150\,\text{MPa}$

Force in tension steel:

$T := f_s \cdot A_s \qquad T = 420\,\text{kN}$

Maximum strain and stress in concrete:

$$\frac{\varepsilon_c}{c} = \frac{\varepsilon_s}{d-c} \qquad \varepsilon_c = \frac{\varepsilon_s}{(d-c)} \cdot c \qquad f_c = \varepsilon_c \cdot E_c = E_c \cdot \left[\varepsilon_s \cdot \left(\frac{c}{d-c}\right)\right]$$

Force in concrete:

$$C_c = \frac{1}{2} \cdot (f_c) \cdot (c \cdot b) = \frac{1}{2} \cdot \left[E_c \cdot \left[\varepsilon_s \cdot \left(\frac{c}{d-c}\right)\right]\right] \cdot (c \cdot b)$$

$$\Sigma F_x = 0 \qquad f_s \cdot A_s = \frac{1}{2} \cdot \left[E_c \cdot \left[\varepsilon_s \cdot \left(\frac{c}{d-c}\right)\right]\right] \cdot (c \cdot b)$$

$$\left(\frac{1}{2} \cdot E_c \cdot \varepsilon_s \cdot b\right) \cdot c^2 + (f_s \cdot A_s) \cdot c - f_s \cdot A_s \cdot d = 0$$

where: $\left(\frac{1}{2} \cdot E_c \cdot \varepsilon_s \cdot b\right) = 3696.644\,\frac{\text{N}}{\text{mm}} \qquad (f_s \cdot A_s) = 420 \times 10^3\,\text{N} \qquad f_s \cdot A_s \cdot d = 228.9 \times 10^6\,\text{N} \cdot \text{mm}$

Solving: $c = 198.43\,\text{mm}$

Check: $\varepsilon_c := \frac{\varepsilon_s}{(d-c)} \cdot c \qquad \varepsilon_c = 0.000429 \qquad f_c := \varepsilon_c \cdot E_c \qquad f_c = 12.095\,\text{MPa}$

$C_c := \frac{1}{2} \cdot (f_c) \cdot (c \cdot b) \qquad C_c = 420\,\text{kN}$ OK

Internal Moment:

$\Sigma M_{Cc} = 0$ $M_n = f_s \cdot A_s \cdot jd$ $jd := d - c + \frac{2}{3} \cdot c$ $jd = 478.9\,mm$

$M_n := f_s \cdot A_s \cdot jd$ $M_n = 201.1\,kN \cdot m$

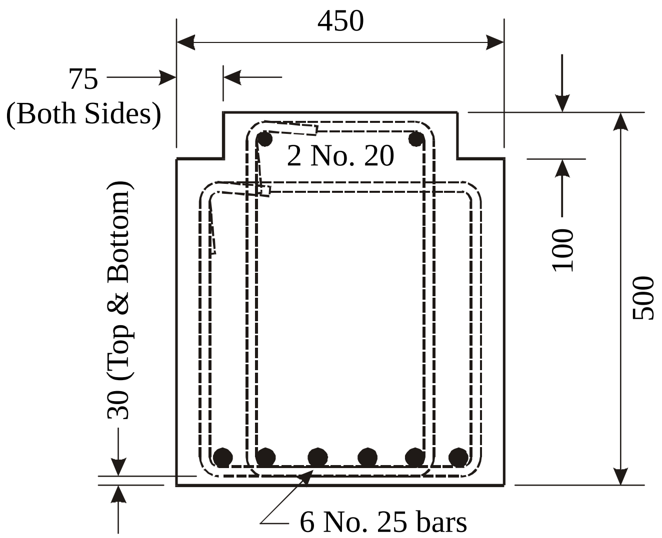
Alternate solution - Integration:

$\varepsilon_c(y) := \left[\varepsilon_s \cdot \left(\frac{y}{d - c}\right)\right]$ $f_c(y) = E_c \cdot \left[\varepsilon_s \cdot \left(\frac{y}{d - c}\right)\right]$

$C_c = \int_0^c f_c(y) \cdot b\,dy$ $\int_0^c E_c \cdot \left[\varepsilon_s \cdot \left(\frac{y}{d - c}\right)\right] \cdot b\,dy$

$C_c = \frac{1}{2} \cdot c^2 \cdot E_c \cdot \frac{\varepsilon_s}{(d - c)} \cdot b$ Same equation as above

Question 2:



- Beam dimensions:

$h := 500 \cdot mm$ $\quad b_{bot} := 450 \cdot mm$

$b_{top} := b_{bot} - 2 \cdot 75 \cdot mm$ $\quad b_{top} = 300\,mm$

$h_{fl} := 100 \cdot mm$

$cc := 30 \cdot mm$ $\quad d_{st} := 10 \cdot mm$

- Tension Steel:

$d_b := 25 \cdot mm$ $\quad n_{bar} := 6$

$A_s := A_{s\_bar}(d_b) \cdot n_{bar}$ $\quad A_s = 3000\,mm^2$

$d := h - cc - d_{st} - 0.5 \cdot d_b$ $\quad d = 447.5\,mm$

- Compression Steel:

$d'_b := 20 \cdot mm$ $\quad n'_{bar} := 2$ $\quad A'_s := A_{s\_bar}(d'_b) \cdot n'_{bar}$ $\quad A'_s = 600\,mm^2$

$d' := cc + d_{st} + 0.5 \cdot d'_b$ $\quad d' = 50\,mm$

- Material: $f'_c := 30 \cdot MPa$ $\quad f_y := 400 \cdot MPa$ $\quad \varepsilon_{cu} := 0.0035$ $\quad E_s := 200000 \cdot MPa$

Whitney stress block parameters: $\alpha_1 = 0.805$ $\quad \beta_1 = 0.895$

- Area of tensile steel to cause compressive stress block to align with bottom of top flange:

$\Sigma F_x = 0$ $\quad C_c + C_s = T$ $\quad a := h_{fl}$ $\quad c := \beta_1 \cdot a$ $\quad c = 89.5\,mm$

$C_c := (\phi_c \cdot \alpha_1 \cdot f'_c) \cdot (a \cdot b_{top})$ $\quad C_c = 434.7\,kN$

Compression steel:

$\varepsilon'_s := \varepsilon_{cu} \cdot \left(\frac{c - d'}{c}\right)$ $\quad \varepsilon'_s = 0.00154$ $\quad < \varepsilon_y$ $\quad f'_s := \varepsilon'_s \cdot E_s$ $\quad f'_s = 308.9\,MPa$

$C_s := A'_s \cdot (\phi_s \cdot f'_s - \phi_c \cdot \alpha_1 \cdot f'_c)$ $\quad C_s = 148.865\,kN$

Tension steel: $\varepsilon_s := \varepsilon_{cu} \cdot \left(\frac{d - c}{c}\right)$ $\quad \varepsilon_s = 0.014$ $\quad > \varepsilon_y$ $\quad f_s := f_y$

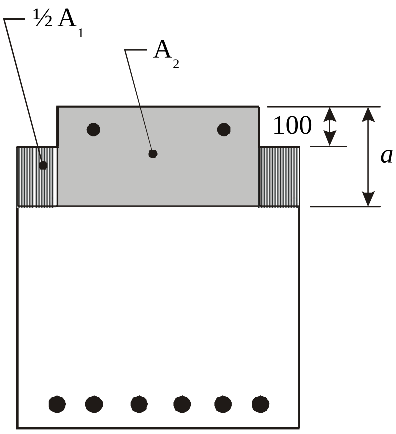
$C_c + C_s = \phi_s \cdot f_s \cdot A_{s\_fl}$

$A_{s\_fl} := \frac{(C_c + C_s)}{(\phi_s \cdot f_s)}$ $\quad A_{s\_fl} = 1716.4\,mm^2$ $\quad < A_s$ - comp. stress block extends into web

- Ultimate moment resistance:

Neutral axis location:

- Assume tension and compression steel yields $f_s := f_y$ $f'_s := f_y$



Compression in concrete:

$$C_{c1} = (\phi_c \cdot \alpha_1 \cdot f'_c) \cdot [(b_{bot} - b_{top}) \cdot (a - h_{fl})]$$

$$C_{c2} = (\phi_c \cdot \alpha_1 \cdot f'_c) \cdot (b_{top} \cdot a)$$

Compression steel:

$$C_s := A'_s \cdot (\phi_s \cdot f'_s - \phi_c \cdot \alpha_1 \cdot f'_c) \qquad C_s = 195.306\ \text{kN}$$

Tension Steel:

$$T := \phi_s \cdot f_s \cdot A_s \qquad T = 1020\ \text{kN}$$

$$\Sigma F_x = 0 \qquad C_c + C_s = T$$

$$(\phi_c \cdot \alpha_1 \cdot f'_c) \cdot [(b_{bot} - b_{top}) \cdot (a - h_{fl})] + (\phi_c \cdot \alpha_1 \cdot f'_c) \cdot (b_{top} \cdot a) + A'_s \cdot (\phi_s \cdot f'_s - \phi_c \cdot \alpha_1 \cdot f'_c) = \phi_s \cdot f_s \cdot A_s$$

$$a := \frac{-(-\phi_c \cdot \alpha_1 \cdot f'_c \cdot h_{fl} \cdot b_{bot} + \phi_c \cdot \alpha_1 \cdot f'_c \cdot h_{fl} \cdot b_{top} + A'_s \cdot \phi_s \cdot f'_s - A'_s \cdot \phi_c \cdot \alpha_1 \cdot f'_c - \phi_s \cdot f_s \cdot A_s)}{(\phi_c \cdot \alpha_1 \cdot f'_c \cdot b_{bot})}$$

$a = 159.8\ \text{mm}$ > 100 mm - OK

$$c := \frac{a}{\beta_1} \qquad c = 178.6\ \text{mm}$$

Check steel yielding:

Tension steel: $\varepsilon_s := \varepsilon_{cu} \cdot \left(\frac{d - c}{c}\right)$ $\varepsilon_s = 0.005272$ $> \varepsilon_y$ - OK

Compression Steel: $\varepsilon'_s := \varepsilon_{cu} \cdot \left(\frac{c - d'}{c}\right)$ $\varepsilon'_s = 0.00252$ $> \varepsilon_y$ - OK

Resisting Moment $\Sigma M_T := 0$ Moment arm:

$C_{c1} := (\phi_c \cdot \alpha_1 \cdot f'_c) \cdot [(b_{bot} - b_{top}) \cdot (a - h_{fl})]$ $C_{c1} = 130\ \text{kN}$ $jd_{c1} := d - a + \frac{a - h_{fl}}{2}$ $jd_{c1} = 317.59\ \text{m}$

$C_{c2} := (\phi_c \cdot \alpha_1 \cdot f'_c) \cdot (b_{top} \cdot a)$ $C_{c2} = 694.7\ \text{kN}$ $jd_{c2} := d - \frac{a}{2}$ $jd_{c2} = 367.59\ \text{mm}$

$C_s := A'_s \cdot (\phi_s \cdot f'_s - \phi_c \cdot \alpha_1 \cdot f'_c)$ $C_s = 195.306\ \text{kN}$ $jd_s := d - d'$ $jd_s = 397.5\ \text{mm}$

$C_{c1} + C_{c2} + C_s - T = 0\ \text{N}$ OK

$M_r := C_{c1} \cdot jd_{c1} + C_{c2} \cdot jd_{c2} + C_s \cdot jd_s$ $M_r = 374.3\ \text{kN} \cdot \text{m}$

Question 3:

- Material: $f'_c := 30 \cdot MPa$ $f_y := 400 \cdot MPa$ $\varepsilon_{cu} := 0.0035$ $E_s := 200000 \cdot MPa$

Whitney stress block parameters: $\alpha_1 = 0.805$ $\beta_1 = 0.895$

- Dimensions:

Beam: $h := 450 \cdot mm$ $b := 400 \cdot mm$ $cc := 40 \cdot mm$ $d_{st} := 10 \cdot mm$

$l_{n\_1} := 7000 \cdot mm$ $l_{n\_2} := 8000 \cdot mm$

Column width: $w_{col} := 400 \cdot mm$

Frame spacing (clear): $B := 3000 \cdot mm$ $B_{cant} := 1500 \cdot mm$

Roof:

Concrete topping: $A_{ct} := 0.10 \cdot m^2$ per m width

Snow load: $q_s := 2.0 \cdot kPa$

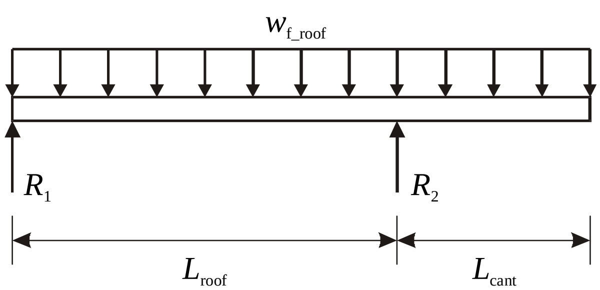
Solution:

Load from roof: Take unit design strip of roof (spanning 1-way from back to front of garage)

Roof self weight: $w_{Dr} := A_{ct} \cdot (\gamma_c \cdot g)$ $w_{Dr} = 2.35 \frac{kN}{m}$

Snow load: $w_{Lr} := q_s \cdot (1.0 \cdot m)$ $w_{Lr} = 2 \frac{kN}{m}$

$w_{f\_roof} := \alpha_D \cdot w_{Dr} + \alpha_L \cdot w_{Lr}$ $w_{f\_roof} = 5.94 \frac{kN}{m}$ per m width of roof



Simple span lengths (c-c of supports)

$L_{roof} := B + 2 \cdot (0.5 \cdot w_{col})$

$L_{roof} = 3400\,mm$

$L_{cant} := B_{cant} + 0.5 \cdot w_{col}$

$L_{cant} = 1700\,mm$

$$\Sigma M_{R1} = 0 \qquad -R_2 \cdot L_{roof} + w_{f\_roof} \cdot \frac{(L_{roof} + L_{cant})^2}{2} = 0$$

$$R_2 := \frac{1}{2} \cdot w_{f\_roof} \cdot \frac{(L_{roof} + L_{cant})^2}{L_{roof}} \qquad R_2 = 22.73\,kN$$

Load on Front Concrete Beam:

Load from roof: $w_{fr} := \dfrac{R_2}{1.0 \cdot m}$ $\quad w_{fr} = 22.728 \dfrac{kN}{m}$

Beam self weight: $w_{Db} := (b \cdot h) \cdot (\gamma_c \cdot g)$ $\quad w_{Db} = 4.236 \dfrac{kN}{m}$

Total factored beam load: $w_f := \alpha_D \cdot w_{Db} + w_{fr}$ $\quad w_f = 28.024 \dfrac{kN}{m}$

Negative support moment: $l_n := 0.5\left(l_{n\_1} + l_{n\_2}\right)$ $\quad l_n = 7.5\,m$

$M_{f\_neg} := \dfrac{-w_f \cdot l_n^{\,2}}{9}$ $\quad M_{f\_neg} = -175.148\,kN \cdot m$ $\quad M_f := -M_{f\_neg}$

Assume No. 30 bars: $d_b := 30 \cdot mm$ $\quad d := h - cc - d_{st} - \dfrac{d_b}{2}$ $\quad d = 385\,mm$

$K_r := \dfrac{M_f}{b \cdot d^2}$ $\quad K_r = 2.954\,MPa$

Also: $K_r = \phi_s \cdot \rho \cdot f_y \cdot \left(1 - \dfrac{\phi_s \cdot \rho \cdot f_y}{2 \cdot \phi_c \cdot \alpha_1 \cdot f'_c}\right)$

$$\rho(K_r) := \frac{\left[\phi_c \cdot \alpha_1 \cdot f'_c - \left(\phi_c^{\,2} \cdot \alpha_1^{\,2} \cdot f'^{\,2}_c - 2 \cdot K_r \cdot \phi_c \cdot \alpha_1 \cdot f'_c\right)^{\left(\frac{1}{2}\right)}\right]}{\left(f_y \cdot \phi_s\right)}$$

$\rho(K_r) = 0.00982$

$A_{s\_req} := \rho(K_r) \cdot b \cdot d$ $\quad A_{s\_req} = 1512.3\,mm^2$

Try: 3 No. 25 bars $\quad d_b := 25 \cdot mm$ $\quad n_{bar} := 3$ $\quad A_s := A_{s\_bar}(d_b) \cdot n_{bar}$ $\quad A_s = 1500\,mm^2$

Check yielding:

$\rho := \dfrac{A_s}{b \cdot d}$ $\quad \rho_b = 0.02427$ $\quad$ >> $\quad \rho = 0.00974$ $\quad$ OK

or (alternate method):

$a := \phi_s \cdot A_s \cdot \dfrac{f_y}{\left(\phi_c \cdot \alpha_1 \cdot f'_c \cdot b\right)}$ $\quad a = 87.99\,mm$

$$c := \frac{a}{\beta_1} \qquad c = 98.31\,mm$$

$$\frac{c}{d} = 0.255 \qquad \frac{700 \cdot MPa}{700 \cdot MPa + f_y} = 0.64 \qquad \text{OK}$$

Check beam width: $aggregate := 20 \cdot mm$ Clear cover: $cc = 40\,mm$

Clear spacing: $s_1 := 1.4 \cdot d_b$ $s_1 = 35\,mm$ <--- Governs

$s_2 := 1.4 \cdot aggregate$ $s_2 = 28\,mm$

$s_3 := 30 \cdot mm$

$s := s_1$

$$b_{req} := 2 \cdot (cc + d_{st}) + (n_{bar}) \cdot d_b + (n_{bar} - 1) \cdot s \qquad b_{req} = 245\,mm \quad < \quad b = 400\,mm \quad \text{OK}$$

Check minimum steel area:

$$A_{smin} := 0.2 \cdot MPa \cdot \frac{\sqrt{\frac{f'_c}{MPa}}}{f_y} \cdot b \cdot h \qquad A_{smin} = 493\,mm^2 \qquad \text{OK}$$

Therefore, use 3 No. 25 bars for negative reinforcement at central support.

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